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A first-order formalism for timelike and spacelike brane solutions

Bert Janssen
Departamento de Física Teórica y del Cosmos
and Centro Andaluz de Física de Partículas Elementales,
Universidad de Granada, 18071 Granada, Spain
E-mail: bjanssen@ugr.es

Paul Smyth
II. Institut für theoretische Physik der Universität Hamburg,
Luruper Chaussee 149, 22761 Hamburg, Germany
E-mail: Paul.Smyth@desy.de

Thomas Van Riet and Bert Vercnocke
Institute for Theoretical Physics, K.U. Leuven,
Celestijnenlaan 200D, B-3001 Leuven, Belgium
E-mail: Bert.Vercnocke@fys.kuleuven.be, Thomas.VanRiet@fys.kuleuven.be

Abstract: We show that the construction of BPS-type equations for non-extremal black holes due to Miller et. al. can be extended to branes of arbitrary dimension and, more importantly, to time-dependent solutions. We call these first-order equations *fake*- or *pseudo*-BPS equations in light of the formalism that has been developed for domain wall and cosmological solutions of gravity coupled to scalar fields. We present the fake/pseudo-BPS equations for all stationary branes (timelike branes) and all time-dependent branes (spacelike branes) of an Einstein-dilaton-p-form system in arbitrary dimensions.

Keywords: p-branes, Black Holes in String Theory, Black Holes, Classical Theories of Gravity.

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1. Introduction

Many brane-type solutions have been constructed for simple truncations of supergravity theories to the form (see, for instance, [1])

$$S = \int \mathrm{d}^D x\sqrt{-g}\Big(\frac{1}{2\kappa_D^2}\mathcal{R} - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2n!}\mathrm{e}^{a\phi}F_n^2\Big)\,, \tag{1.1}$$

where $\mathcal{R}$ is the Ricci-scalar, κ_D^2 is the D-dimensional gravitational coupling, ϕ is a scalar field (the dilaton) and F_n is the field strength of some $(n-1)$-form, $F_n = \mathrm{d}A_{n-1}$ if $n > 0$. For the special case $n = 0$ we consider F_n^2 to be a cosmological term (scalar potential). The parameter a is fixed and is called the dilaton coupling.

The equations of motion derived from this action admit electrically charged $(n-2)$-branes and magnetically charged $(D-n-2)$-branes. A brane solution can be stationary or time-dependent. The metric of a stationary p-brane is given by

$$\mathrm{d}s_D^2 = \mathrm{e}^{2A(r)}\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + \mathrm{e}^{2B(r)}\mathrm{d}r^2 + \mathrm{e}^{2C(r)}\mathrm{d}\Sigma_k^2\,, \tag{1.2}$$

where η is the usual Minkowski metric in $p+1$ dimensions, $\eta = \mathrm{diag}(-,+,\dots,+)$ and $\mathrm{d}\Sigma_k^2$ is the metric of a d-dimensional maximally symmetric space with unit curvature $k = -1, 0, 1$, such that the Ricci scalar is given by $\mathcal{R}_d = kd(d-1)$. When $k = 1$ the solutions possess a rotational symmetry and can be asymptotically flat (in contrast to $k = -1$). For $D = 10$ and specific values of a and n the solutions correspond to D-branes in string theory.

The metric of the time-dependent branes is similar

$$\mathrm{d}s_D^2 = \mathrm{e}^{2A(t)}\delta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu - \mathrm{e}^{2B(t)}\mathrm{d}t^2 + \mathrm{e}^{2C(r)}\mathrm{d}\Sigma_k^2\,, \tag{1.3}$$

where δ is the usual flat Euclidean metric in $p+1$ dimensions, $\delta = \mathrm{diag}(+,+\ldots+)$. In the $k=-1$ case the transverse space possesses a Lorentzian symmetry and can be asymptotically flat (in contrast to $k=+1$ solutions). These solutions are the spacelike branes (S-branes) introduced by Gutperle and Strominger [2], who conjectured that such branes correspond to specific time-dependent processes in string theory.

From now on we shall call the stationary branes with spherical slicing ($k=+1$) *timelike* branes and the time-dependent branes with hyperbolic slicing ($k=-1$) *spacelike* branes. All the other possible slicings are also covered here, but we choose to highlight only these two cases.

It has been known for a long time that particular timelike p-brane solutions of supergravity preserve some fraction of supersymmetry. Practically this means that the solutions fulfill some *first-order* differential equations that arise from demanding the supersymmetry transformations to be consistently satisfied for vanishing fermions. Such first-order equations have become known as Bogomol'nyi or BPS equations, after Bogomol'nyi's [3], and Prasad and Sommerfield's [4] work on first-order equations and exact solutions for magnetic monopoles in the Yang-Mills-Higgs theory. It was then later shown that this limit is intimately linked to the preserved supersymmetry of solitons in supersymmetric theories by Witten and Olive [5]. The term *BPS equation* is now generically used for equations of motion that are inferred by rewriting the action as a sum of squares. Supersymmetric solutions, in general, belong to this class. Stationary non-extremal and time-dependent solutions cannot preserve supersymmetry in ordinary supergravity theories. Naively one therefore expects that such solutions cannot be found from BPS equations, but rather by solving the full second-order equations of motion.

To our knowledge there are three instances in the literature where it has been shown that this view is too pessimistic:

1. Not all extremal black hole solutions of supergravity have to be supersymmetric. It turns out that many non-supersymmetric but extremal solutions fulfill first-order equations in a given supergravity theory (see for instance [6–8]). More surprisingly, Miller et. al have shown that the *non-extremal* Reissner-Nordström black hole solution of Einstein-Maxwell theory can be found from first-order equations [9] by a clever rewriting of the action as a sum of squares à la Bogomol'nyi. The method of [9] is the main tool for the present paper.

2. Many stationary domain wall solutions that do not preserve any supersymmetry have been shown to allow for first order-equations by the construction of a *fake* superpotential [10–12]. The domain walls in question are solutions to the following Lagrangian

$$\mathcal{L} = \sqrt{-g}\Big(\mathcal{R} - \frac{1}{2}G_{ij}(\Phi)g^{\mu\nu}\partial_\mu\Phi^i\partial_\nu\Phi^j - V(\Phi)\Big) \tag{1.4}$$

where Φ^i are scalars fields, $G_{ij}(\Phi)$ is the metric on the target space and $V(\Phi)$ is the scalar potential. The metric Ansatz for a flat domain wall is[1]

$$\mathrm{d}s^2 = \mathrm{e}^{2B(z)}\mathrm{d}z^2 + \mathrm{e}^{2A(z)}\eta_{ab}\mathrm{d}x^a\mathrm{d}x^b\,, \tag{1.5}$$

where η_{ab} is diag$(-,+,\dots,+)$. The high degree of symmetry of this Ansatz is only consistent when the fields that support the solution depend solely on the z-coordinate i.e. $\Phi^i = \Phi^i(z)$. We then suppose that a scalar function $W(\Phi)$ exists such that

$$V = \frac{1}{2}G^{ij}\partial_i W\partial_j W - \frac{D-1}{4(D-2)}W^2\,, \tag{1.6}$$

which allows the action to be written as a sum of squares (neglecting boundary terms) [13]

$$S = \int \mathrm{d}z\,\mathrm{e}^{(D-1)A+B}\left\{\frac{(D-1)}{4(D-2)}\big[W - 2(D-2)\mathrm{e}^{-B}A'\big]^2 - \frac{1}{2}\Big\|\mathrm{e}^{-B}(\Phi^i)' + G^{ij}\partial_j W\Big\|^2\right\}, \tag{1.7}$$

where a prime denotes a derivative with respect to z. Solutions are obtained when each square in the action is zero. If W is a superpotential of some supersymmetric theory, these first-order equations are the standard BPS equations for domain walls that would arise by demanding that the supersymmetry variations are satisfied for vanishing fermions. However, for *every* W that obeys (1.6) we can find a corresponding DW-solution. If W is not related to the quantity appearing in the supersymmetry transformations the resulting solutions are called *fake supersymmetric.*

3. FLRW-cosmologies are very similar to domain walls [14–16], the difference in metrics being given by a few signs

$$\mathrm{d}s^2 = -\mathrm{e}^{2B(t)}\mathrm{d}t^2 + \mathrm{e}^{2A(t)}\delta_{ab}\mathrm{d}x^a\mathrm{d}x^b\,. \tag{1.8}$$

When the relation (1.6) is changed by an overall minus-sign

$$V = -\frac{1}{2}G^{ij}\partial_i W\partial_j W + \frac{D-1}{4(D-2)}W^2\,, \tag{1.9}$$

the same first-order equations for domain walls exist for cosmologies, where now the primes indicate derivatives with respect to time. These relations have become known as *pseudo-BPS* conditions [15, 16] (see also [17, 18] for the first-order framework in cosmology). As for domain walls one readily checks that these first-order equations arise from the fact that the action can be written as a sum of squares [19]. The structure underlying the existence of these first-order equations can be understood from Hamilton-Jacobi theory [20–22].[2]

[1]For simplicity we only discuss flat cosmologies and flat domain walls.

[2]In ordinary supergravity theories the pseudo-BPS relations cannot be related to supersymmetry preservation. However, in the case of supergravity theories with 'wrong sign' kinetic terms the pseudo-BPS relations are related to true supersymmetry [23–27]. In this paper we shall consider ordinary supergravity theories and therefore pseudo-BPS conditions are not related to supersymmetry. Practically this means that we have first-order equations which can be understood to originate from a Bogomol'nyi rewriting of the action.

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The examples given above (1-3) are only a subset of the different p-branes that exist, namely timelike 0-branes in $D = 4$ (the RN black holes) and $(D-2)$-branes (domain walls and FLRW-cosmologies). It is the aim of this paper to understand in general when stationary and time-dependent p-branes in arbitrary dimensions can be found from BPS equations.

One of the main subtleties that arises in this generalisation is that there exist two kinds of black deformations of timelike p-branes which coincide for the special case of black holes in four dimensions. For this reason the example treated by Miller et. al. [9] is not completely representative. Secondly, for time-dependent solutions, it has yet to be understood if the concept of pseudo-supersymmetry could be extended beyond cosmologies (time-dependent $(D-2)$-branes) to general time-dependent p-branes (see [25, 27] for initial progress in this direction).

The rest of the paper is organized as follows. In section 2 we consider Einstein-Maxwell theory and repeat the construction of the first-order equations for the non-extremal Reissner-Nordström black hole. We immediately show that the same technique allows one to rederive the S0-brane solution of Einstein-Maxwell theory [2]. In section 3 we discuss the special case of (-1)-branes in arbitrary dimensions. In section 4 we explain how the BPS equations for the (-1)-branes imply the BPS equations for general p-branes in arbitrary dimensions via an uplifting procedure. We then discuss the issue of different black deformations in section 5 and finish with conclusions in section 6.

2. Four-dimensional Einstein-Maxwell theory

Einstein-Maxwell theory in four dimensions is described by the action

$$S = \int \mathrm{d}^4x\sqrt{-g}\Big(\frac{1}{2\kappa^2}\mathcal{R} - \frac{1}{4}F^2\Big)\,, \tag{2.1}$$

and has electric and magnetic 0-branes solutions. Following [9] we shall choose a particular Ansatz for the 0-brane metric which turns out to be useful

$$\mathrm{d}s^2 = -\epsilon \mathrm{e}^{2A(u)}\mathrm{d}z^2 + \mathrm{e}^{-2A(u)+2B(u)}\Big(\epsilon \mathrm{e}^{2C(u)}\mathrm{d}u^2 + \mathrm{d}\Sigma_k^2\Big)\,. \tag{2.2}$$

If $\epsilon = +1$ then z is time $z = t$ and the metric is static. For spherical slicings $(k = +1)$ this is the appropriate Ansatz for a black hole, where u is then some function of the familiar radial coordinate r. When $\epsilon = -1$ the metric is time-dependent and for hyperbolic slicings $(k = -1)$ this is the appropriate Ansatz for an S0-brane [2] with a one-dimensional Euclidean worldvolume labelled by z, and u is some function of the time-coordinate τ used in the Milne patch of Minkowski spacetime. The general Ricci scalar is given by

$$\mathcal{R} = 2\,\epsilon\,\mathrm{e}^{2(A-B-C)}\Big(\ddot{A} - \dot{A}^2 + \dot{A}\dot{B} - \dot{A}\dot{C} - 2\ddot{B} + 2\dot{B}\dot{C} - \dot{B}^2\Big) + 2k\mathrm{e}^{2(A-B)}\,, \tag{2.3}$$

where a dot indicates a derivative with respect to u.

For *electrical* solutions, the Maxwell and Bianchi equations are solved by

$$F_{uz} = Q\mathrm{e}^{2A-B+C}\,. \tag{2.4}$$

Plugging the Ansätze (2.2) and (2.4) into the Einstein field equations derived from (2.1), one can ask whether the resulting second-order equations of motion in the one variable u can be interpreted as field equations for A, B and C derived from a one-dimensional effective action. It is straightforward to see that the equations of motion can be obtained by varying the following action

$$S = \int \mathrm{d}u\ \mathrm{e}^{B-C}\left(2\dot{B}^2 - 2\dot{A}^2 + 2\epsilon\, k\mathrm{e}^{2C} - \epsilon\,\kappa^2 Q^2 \mathrm{e}^{2(A-B+C)}\right). \tag{2.5}$$

This action cannot be obtained from direct substitution of the Ansätze into the four-dimensional action as the sign of the resulting Q^2-term would be wrong. This sign discrepancy does not appear for purely magnetic solutions, for which the Ansätze can be plugged into the action consistently. We discuss this point in detail in appendix A and refer to [28] for a careful derivation of the black hole effective action in a more general setting.

The field C does not appear with a derivative in the action and is therefore not a propagating degree of freedom. This was to be expected since C is related to the reparametrization freedom of u. The field C acts as a Lagrange multiplier enforcing the following constraint

$$2\dot{B}^2 - 2\dot{A}^2 - 2\epsilon\, k\mathrm{e}^{2C} + \epsilon\,\kappa^2 Q^2 \mathrm{e}^{2(A-B+C)} = 0\,. \tag{2.6}$$

As long as this contraint is satisfied we are free to pick a gauge choice for C as we like. In the following we choose the gauge $B = C$.

It turns out that it is easy to generalize the Bogomol'nyi bound found in [9] to include both stationary and time-dependent configurations with arbitrary slicing of the transverse space $k = 0, \pm 1$. The action (2.5) is, up to total derivatives, equivalent to

$$S = \int \mathrm{d}u\ 2\left(\dot{B} + \sqrt{\epsilon k\mathrm{e}^{2B} + \beta_1^2}\right)^2 - 2\left(\dot{A} + \sqrt{\epsilon\frac{\kappa^2}{2}Q^2\mathrm{e}^{2A} + \beta_2^2}\right)^2, \tag{2.7}$$

where β_1 and β_2 are constants. The BPS equations are

$$\dot{B} = -\sqrt{\epsilon k\mathrm{e}^{2B} + \beta_1^2}\,, \qquad \dot{A} = -\sqrt{\epsilon\frac{\kappa^2}{2}Q^2\mathrm{e}^{2A} + \beta_2^2}\,. \tag{2.8}$$

The constraint (2.6) implies that $\beta_1^2 = \beta_2^2 \equiv \beta^2$. Note that for time-dependent solutions with charge ($\epsilon = -1, \quad Q \neq 0$) the limit of $\beta \to 0$ does not exist, while for $Q = 0$ the limit only exists for $k = -1$. The BPS equations are all of the form

$$\dot{D}_\pm = -\sqrt{\beta^2 \pm K^2\mathrm{e}^{2D_\pm}}\,, \tag{2.9}$$

where K is a constant, depending on the case under consideration. The solutions to these equations are given by

$$\mathrm{e}^{-D_+} = \frac{K}{\beta}\sinh(\beta u + c_+)\,, \qquad \mathrm{e}^{-D_-} = \frac{K}{\beta}\cosh(\beta u + c_-)\,, \tag{2.10}$$

where $c_\pm$ are constants of integration. In the extremal limit $\beta \to 0$ the solution becomes $\mathrm{e}^{-D_+} = \mathrm{e}^{-D_-} = Ku + c$.

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Rediscovering Reissner-Nordström black holes. For the black hole Ansatz ($\epsilon = +1\,, k = +1$) it was shown in [9] that solving the BPS equation described above leads to the non-extremal Reissner-Nordström solutions. We shall now quickly review this for completeness, and draw attention to some further subtleties.

The solutions of the first-order equations (2.8) are

$$\mathrm{e}^{-A} = \frac{Q\kappa}{\sqrt{2}\beta}\sinh(\beta u + c_a)\,, \qquad \mathrm{e}^{-B} = \frac{1}{\beta}\sinh\beta u\,, \tag{2.11}$$

where we put the integration constant in the solution for B to zero by shifting the origin of the u-axis, leading to the following metric

$$\mathrm{d}s^2 = -\frac{2\beta^2}{Q^2\kappa^2\sinh^2(\beta u + c_a)}\mathrm{d}t^2 + \frac{\beta^2\kappa^2 Q^2\sinh^2(\beta u + c_a)}{2\sinh^4\beta u}\mathrm{d}u^2 + \frac{\kappa^2 Q^2\sinh^2(\beta u + c_a)}{2\sinh^2\beta u}\mathrm{d}\Omega_2^2\,. \tag{2.12}$$

We can identify the radial coordinate r^2 as the function in front of $\mathrm{d}\Omega_2^2$. In order to obtain the standard form of the Reissner-Nordström solution, one has to perform the following coordinate transformation:

$$r = \frac{\kappa Q\sinh(\beta u + c_a)}{\sqrt{2}\sinh\beta u}\,, \qquad \tau = \frac{\sqrt{2}\beta}{\kappa Q\sinh c_a}t\,, \tag{2.13}$$

such that the solution takes the form

$$\mathrm{d}s^2 = -H(r)\mathrm{d}\tau^2 + H(r)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega_2^2\,, \qquad F_{\tau r} = -\frac{Q}{r^2}, \tag{2.14}$$

where

$$H(r) = 1 - \frac{2\kappa Q\cosh c_a}{\sqrt{2}r} + \frac{\kappa^2 Q^2}{2r^2}. \tag{2.15}$$

It is then clear that the ADM mass corresponds to $M = \kappa Q\cosh c_a/\sqrt{2}$. Note that for $\cosh c_a = 1$, the above solution reduces to the extreme Reissner-Nordström metric, implying that $\cosh c_a$ is related to the non-extremality parameter β. Indeed, from (2.11) we have that

$$\cosh c_a = \sqrt{1 + \frac{2\beta^2\mathrm{e}^{-2A(0)}}{\kappa^2 Q^2}}, \tag{2.16}$$

such that the limit $\cosh c_a = 1$ corresponds to $\beta = 0$, as one expects from the action (2.7).

Rediscovering spacelike 0-branes. For spacelike branes ($\epsilon = k = -1$) we find

$$\mathrm{e}^{-A} = \frac{\kappa Q}{\sqrt{2}\beta}\cosh(\beta u + c_a)\,, \qquad \mathrm{e}^{-B} = \frac{1}{\beta}\sinh(\beta u)\,. \tag{2.17}$$

Once again, shifting the origin of the u-axis, the integration constant in the equation for B has been put to zero. Using the coordinate transformation

$$\tau = \frac{\kappa Q\cosh(\beta u + c_a)}{\sqrt{2}\sinh\beta u}\,, \qquad x = \frac{\sqrt{2}\beta}{\kappa Q\cosh c_a}z\,, \tag{2.18}$$

the solution then takes the following form

$$\mathrm{d}s^2 = G(\tau)\,\mathrm{d}x^2 - G(\tau)^{-1}\mathrm{d}\tau^2 + \tau^2\mathrm{d}\mathbb{H}_2^2\,, \qquad F_{\tau x} = \frac{Q}{\tau^2}\,, \tag{2.19}$$

with

$$G(\tau) = 1 - 2\frac{\sinh(c_a)\kappa Q}{\sqrt{2}\tau} - \frac{Q^2\kappa^2}{2\tau^2}\,, \tag{2.20}$$

where we introduced the metric for the hyperboloid $\mathrm{d}\mathbb{H}_2^2 = \mathrm{d}\Sigma_{-1}^2$. Again, this solution is asymptotically flat. Moreover, we see that this reduces to the metric for the S0-brane of [2] after a constant rescaling of x and τ. Taking the limit $\beta \to 0$, the metric is easily seen to describe flat space in Milne coordinates.

Addition of a dilaton. Before we proceed to the case of p-branes in arbitrary dimensions let us first consider the coupling of the vector field to a dilaton, as this is the generic situation in supergravity theories. The action describing four-dimensional Einstein-Maxwell-dilaton theory is

$$S = \int \mathrm{d}x^4\sqrt{-g}\left(\frac{1}{2\kappa^2}\mathcal{R} - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}\mathrm{e}^{a\phi}F^2\right). \tag{2.21}$$

The Ansatz for electrical solutions is now given by $F_{uz} = Q\mathrm{e}^{2A-B+C-a\phi}$. In the gauge $B = C$ the effective action becomes

$$S = \int \mathrm{d}u\ 2\dot{B}^2 - 2\dot{A}^2 - \kappa^2\dot{\phi}^2 + 2\epsilon\, k\mathrm{e}^{2B} - \epsilon\,\kappa^2 Q^2\mathrm{e}^{2A-a\phi}\,. \tag{2.22}$$

It turns out to be convenient to define new variables A_1 and ϕ_1

$$A_1 = A - \frac{a}{2}\phi\,, \qquad \phi_1 = \frac{a}{\kappa^2}A + \phi\,. \tag{2.23}$$

With these new variables the Bogomol'nyi form is obvious and similar to the previous case without a dilaton. Writing the action as a sum of squares, we now introduce three constants β_1, β_2 and β_3

$$S = \int \mathrm{d}u\ 2\Big(\dot{B}+\sqrt{\epsilon k\mathrm{e}^{2B} + \beta_1^2}\Big)^2 - \frac{2}{\Delta}\left(\dot{A}_1 + \sqrt{\epsilon\Delta\frac{\kappa^2}{2}Q^2\mathrm{e}^{2A_1} + \beta_2^2}\right)^2 - \frac{\kappa^2}{\Delta}(\dot{\phi}_1 - \beta_3)^2, \tag{2.24}$$

where $\Delta = 1 + (a^2/2\kappa^2)$.

In this case the equivalent of the constraint (2.6) implies that only two of the three integration constants are independent

$$2\beta_1^2 - \frac{2}{\Delta}\beta_2^2 - \frac{\kappa^2}{\Delta}\beta_3^2 = 0\,. \tag{2.25}$$

The BPS equations are the same as before apart from the extra equation $\dot{\phi}_1 = \beta_3$.

When the solutions for A, B and ϕ are plugged into the Ansatz one reproduces the familiar dilatonic black hole solution [29]. One then also notices that the two independent β-parameters appear in a fixed combined way as to effectively form one deformation parameter.

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3. (-1)-branes in D dimensions

A (-1)-brane couples electrically to a 0-form gauge potential, χ, known as the axion. The worldvolume is zero-dimensional and, in the case of a timelike (-1)-brane, this implies that the whole space is Euclidean since it is entirely transverse. The action is

$$S = \int \mathrm{d}x^D \sqrt{-g}\Big(\mathcal{R} - \frac{1}{2}(\partial\phi)^2 + \epsilon\frac{1}{2}\mathrm{e}^{b\phi}(\partial\chi)^2\Big)\,. \tag{3.1}$$

Note the 'wrong sign' kinetic term for the axion when $\epsilon = +1$, which is normal for Euclidean theories. The (-1)-brane Ansatz is

$$\mathrm{d}s_D^2 = \epsilon\mathrm{e}^{2C(z)}\mathrm{d}z^2 + \mathrm{e}^{2A(z)}\mathrm{d}\Sigma_k^2\,, \qquad \phi = \phi(z)\,, \qquad \chi = \chi(z)\,. \tag{3.2}$$

If we consider the axion equation of motion, $\partial_\mu(\sqrt{-g}g^{\mu\nu}\mathrm{e}^{b\phi}\partial_\nu\chi) = 0$, then the solution is of the form

$$\dot{\chi} = Q\ \mathrm{e}^{-b\phi}\,. \tag{3.3}$$

The one-dimensional effective action that reproduces the equations of motion for A and ϕ is

$$S = \int \mathrm{d}z \frac{(D-1)(D-2)}{\kappa^2}\Big(\dot{A}^2\mathrm{e}^{(D-1)A-C} + \epsilon k\mathrm{e}^{(D-3)A+C}\Big) - \mathrm{e}^{(D-1)A-C}\dot{\phi}^2 - \epsilon\mathrm{e}^{C-(D-1)A-b\phi}Q^2\,. \tag{3.4}$$

As we discussed before this form differs from that obtained by direct substitution of the Ansatz into the original action (appendix A). The field C is not propagating and we can choose it at will; the gauge $C = (D-1)A$ is obviously useful. As before we must consider the constraint that arises from varying the action with respect to C. In this gauge, the BPS form of the action is then equal to

$$S = \int \mathrm{d}z\ \frac{(D-1)(D-2)}{\kappa^2}\Big(\dot{A} + \sqrt{\epsilon k\mathrm{e}^{2(D-2)A} + \beta_1^2}\Big)^2 - \Big(\dot{\phi} - \sqrt{\epsilon Q^2\mathrm{e}^{-b\phi} + \beta_2^2}\Big)^2\,, \tag{3.5}$$

supplemented with the constraint

$$\frac{(D-1)(D-2)}{\kappa^2}\Big(\dot{A}^2 - \epsilon k\mathrm{e}^{-2(D-2)A}\Big) - \dot{\phi}^2 + \epsilon\mathrm{e}^{-b\phi}Q^2 = 0. \tag{3.6}$$

The constraint equation tells us that there is only one effective deformation parameter since

$$\beta_2^2 = \frac{(D-1)(D-2)}{\kappa^2}\beta_1^2\,. \tag{3.7}$$

We now first solve the BPS equations with vanishing deformation parameters for $k = \epsilon = 1$. If we define the coordinate ρ via $\mathrm{d}\rho = -\mathrm{e}^{(D-1)A}\mathrm{d}z$, then the BPS equation, $\dot{A} = -\mathrm{e}^{(D-1)A}$, implies that $\rho = \mathrm{e}^A + c$. Shifting ρ such that $c = 0$ we find that the metric describes the Euclidean plane in spherical coordinates

$$\mathrm{d}s_D^2 = \mathrm{d}\rho^2 + \rho^2\mathrm{d}\Omega_{D-1}^2\,. \tag{3.8}$$

The solutions for the scalar fields are

$$e^{\frac{b}{2}\phi} = -\frac{Qb}{2(D-2)}\,\rho^{-D+2} + e^{\frac{b}{2}\phi_\infty}\,, \qquad \chi = -\frac{2|Q|}{bQ}\,(e^{-\frac{b}{2}\phi} - e^{-\frac{b}{2}\phi_\infty}) + \chi_\infty\,. \tag{3.9}$$

This is indeed the extremal instanton solution, see for instance [30, 31]. For non-zero β the solution becomes (in the frame $C = (D-1)A$)

$$e^{(2-D)A} = \frac{1}{\beta_1}\sinh[(D-2)\beta_1\, z + c_1]\,, \tag{3.10}$$

$$e^{-\frac{b}{2}\phi(z)} = \frac{Q}{\beta_2}\sinh\left(\frac{\beta_2 b}{2}\, z + c_2\right), \qquad \chi(z) = -\frac{2}{bQ}\sqrt{Q^2 e^{-b\phi} + \beta_2^2} + c_3\,, \tag{3.11}$$

where c_1, c_2 and c_3 are arbitrary constants of integration. These solutions correspond to the super-extremal instantons that were constructed in [31, 32].

Finally, the time-dependent S(-1) brane solution (with $k = \epsilon = -1$) that was first constructed in [33] can be rederived (again in the frame $C = (D-1)A$)

$$e^{(2-D)A} = \frac{1}{\beta_1}\sinh[(D-2)\beta_1\, t + c_1]\,, \tag{3.12}$$

$$e^{-\frac{b}{2}\phi(t)} = \frac{Q}{\beta_2}\cosh\left(\frac{\beta_2 b}{2}\, t + c_2\right), \qquad \chi(t) = -\frac{2}{bQ}\sqrt{Q^2 e^{-b\phi} - \beta_2^2} + c_3\,. \tag{3.13}$$

4. *p*-branes in arbitrary dimensions

Let us now consider the following theory in $d = D + p + 1$ dimensions

$$S = \int d^d x\sqrt{-g}\left\{\frac{1}{2\kappa^2}\mathcal{R} - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2(p+2)!}e^{a\phi}F_{p+2}^2\right\}. \tag{4.1}$$

The corresponding p-brane solutions can all be reduced to (-1)-brane solutions in D dimensions via reduction over their flat worldvolumes. Therefore we should be able to reproduce the BPS bounds and the BPS solutions using the (-1)-brane calculation of the previous section.

A typical p-brane Ansatz takes the form

$$\begin{aligned} ds^2 &= e^{2\alpha\varphi(z)}ds_D^2 + e^{2\beta\varphi(z)}\ \eta^{\epsilon}_{mn}dy^m dy^n\,, \qquad \phi = \phi(z)\,,\\ A_{p+1}(z) &= \chi(z)\ dy^1\wedge dy^2\wedge\ldots\wedge dy^{p+1}\,, \end{aligned} \tag{4.2}$$

where ds_D^2 is the D-dimensional metric (3.2) and $\eta^\epsilon = \mathrm{diag}(-\epsilon, +1, \ldots, +1)$. The constants α and β are given by

$$\alpha = \sqrt{\frac{p+1}{2(D+p-1)(D-2)}}\,, \qquad \beta = -\sqrt{\frac{D-2}{2(D+p-1)(p+1)}}\,. \tag{4.3}$$

We now reduce the Ansatz (4.2) over the worldvolume coordinates y, obtaining a lower-dimensional Ansatz of the form (3.2). The equivalent reduction of the action (4.1) leads to the D-dimensional action

$$S = \int d^D x\sqrt{-g}\left(\frac{1}{2\kappa^2}\mathcal{R} - \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}(\partial\phi)^2 + \epsilon\frac{1}{2}e^{a\phi+2(D-2)\alpha\varphi}(\partial\chi)^2\right). \tag{4.4}$$

The effective one-dimensional action for the lower-dimensional solution then contains an extra decoupled dilaton when compared to instanton calculation of the previous section,

$$S = \int \mathrm{d}z \, \frac{(D-1)(D-2)}{\kappa^2}\Big(\dot{A}^2 + \epsilon k \mathrm{e}^{2(D-2)A}\Big) - \dot{\tilde{\phi}}^2 - \dot{\tilde{\varphi}}^2 - \epsilon \mathrm{e}^{-b\tilde{\phi}} Q^2 \,, \tag{4.5}$$

where $b^2 = a^2 + 4(D-2)^2\alpha^2$ and the original scalars φ and ϕ are given by

$$\phi = \frac{1}{b}(a\tilde{\phi} - 2(D-2)\alpha\tilde{\varphi})\,, \qquad \varphi = \frac{1}{b}(2(D-2)\alpha\tilde{\phi} + a\tilde{\varphi})\,. \tag{4.6}$$

Up to total derivatives, the BPS-form of the action is then given by

$$S = \int \mathrm{d}z \, \frac{(D-1)(D-2)}{\kappa^2}\Big(\dot{A} + \sqrt{\epsilon k \mathrm{e}^{2(D-2)A} + \beta_1^2}\Big)^2 - \Big(\dot{\tilde{\phi}} + \sqrt{\epsilon Q \mathrm{e}^{-b\tilde{\phi}} + \beta_2^2}\Big)^2 - (\dot{\tilde{\varphi}} + \beta_3)^2\,, \tag{4.7}$$

where only two of the three deformation parameters β_1, β_2 and β_3 are independent due to the condition coming from the constraint equation:

$$\frac{(D-1)(D-2)}{\kappa^2}\beta_1^2 - \beta_2^2 - \beta_3^2 = 0\,. \tag{4.8}$$

The solutions to the BPS equations for A and $\tilde{\phi}$ can be found in the previous section in equations (3.10)–(3.13), whereas the solution for the extra field $\tilde{\varphi}$ is trivial, $\tilde{\varphi}(z) = -\beta_3 z$. From our Ansatz (4.2) and the field redefinition (4.6) we can immediately read of the timelike and spacelike brane solutions in d dimensions. We do not discuss these solutions as they have been discussed in the literature in numerous places.

5. *p*-branes with type II deformations

In general there are two types of black deformations of extremal p-branes that one can consider [34]. Type I deformations are defined by the metric

$$\mathrm{d}s^2 = \mathrm{e}^{2A(r)}\mathrm{d}\vec{x}_{p+1}^2 + \mathrm{e}^{2B(r)}\big(\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2\big)\,. \tag{5.1}$$

The p-brane is said to have a type I deformation if

$$X \equiv (p+1)A + (D-p-3)B \neq 0\,. \tag{5.2}$$

Since the extremal (-1)-brane geometry is the Euclidean plane we read of that $A = \beta\varphi, B = \alpha\varphi$ such that the relation between α and β (4.3) immediately gives $X = 0$. From the previous section we can infer that the cases with $X \neq 0$ can be obtained by uplifting non-extremal instanton solutions.

However there exist other types of deformations of extremal branes which are not contained in the analysis of the previous section. These deformations are labelled type II and the resulting metric breaks the worldvolume symmetry ISO$(p,1)$ down to $\mathbb{R}\times$ ISO(p)

$$\mathrm{d}s^2 = \mathrm{e}^{2A(r)}\big(-\mathrm{e}^{2f(r)}\mathrm{d}t^2 + \mathrm{d}x^i\mathrm{d}x^i\big) + \mathrm{e}^{2B(r)}\big(\mathrm{e}^{-2f(r)}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2\big)\,, \tag{5.3}$$

where $X = 0$, with X defined in (5.2). For black holes and instantons these two types of deformations coincide.

The approach of writing the effective action as a sum of squares is similar to that of the instanton discussions in the previous section, and is based on dimensionally reducing the brane over its worldvolume. Notice that although some worldvolume symmetries are broken we can still carry out the reduction as the translation symmetry is not broken.

It should be clear that it is possible to reduce type II-deformed branes over their wordvolume if the shape moduli are not all truncated. In order to proceed we shall therefore keep a single shape modulus, denoted by T. It appears as follows in the metric Ansatz

$$\mathrm{d}s^2 = \mathrm{e}^{2\alpha\varphi}\mathrm{d}s_D^2 + \mathrm{e}^{2\beta\varphi}\big(-\mathrm{e}^{-T}\mathrm{d}t^2 + \mathrm{e}^{p^{-1}T}\mathrm{d}x_i\mathrm{d}x^i\big)\,. \tag{5.4}$$

The effective action (4.7) then gets the extra term $-\frac{p+1}{2p}(\dot{T} + \beta_4)^2$ and the corresponding constraint equation implies the following relation amongst the deformation parameters

$$\frac{(D-1)(D-2)}{\kappa^2}\beta_1^2 - \beta_2^2 - \beta_3^2 - \frac{p+1}{2p}\beta_4^2 = 0\,. \tag{5.5}$$

Now the various possibilities of choosing non-zero β's correspond to the possible deformations. If all β's are non-zero we have a solution with combined type I and type II deformations. The purely type II deformed solution can be found by choosing $\beta_2 = \beta_3 = 0$ with the other two β's non-zero. Again as these solutions can be easily found in the literature we will not write them explicitly here.

The message here is twofold. Firstly, we have shown that the various types of deformations of p-branes can be found from the first-order formalism. Secondly, this technique is clearly beneficial in finding various (complicated) black brane solutions, and is simple in comparison with the existing techniques of solving the coupled second-order differential equations.

6. Discussion

In this note we have shown that all known brane-type solutions of an Einstein-dilaton-p-form theory can be found from decoupled first-order equations, thereby extending the results of [9] to arbitrary dimensions and time-dependent cases. By brane-type solutions we mean solutions with a space-time Ansatz given by (1.2) and (1.3). The key point is that these solutions depend on one coordinate and therefore can be constructed from a one-dimensional effective action, as was first discussed for black holes in [35]. If this one-dimensional effective action can be written as sums and differences of squares we arrive at first-order equations à la Bogomol'nyi. That this is possible for some extremal timelike brane solutions was to be expected as they can be seen as supersymmetric solutions when embedded into an appropriate supergravity theory.

In [9] the question was raised as to whether these deformed BPS equations could also be understood from the point of view of supersymmetry. One may imagine that the bosonic Lagrangian (1.1) could be embedded into different (non-standard) supergravity theory for which the non-extremal solutions preserve some fraction of supersymmetry. However, there is in fact an obstruction to even defining Killing spinors which implies that the non-extremal solutions cannot preserve supersymmetry. Of course one should repeat the same calculations of [9] for the case of p-branes with $p > 0$, as well as for type I and type II deformations, but we believe that the same negative answer will be found.

We consider the application of these ideas to time-dependent brane solutions (S-branes) as a less trivial extension of [9]. One possible way to understand why it was to be expected that a similar first-order formalism exists for time-dependent branes stems from the known fact that non-extremal stationary branes can be analytically continued to time-dependent solutions, something that is impossible for extremal branes [34]. As explained in the introduction, this first-order formalism for time-dependent p-branes is the natural generalisation of the so-called pseudo-BPS equations for FLRW-cosmologies [15, 16].

We did not completely exhaust all possible brane solutions in our analysis, as we did not consider branes with co-dimension less than three. When the co-dimension is one, the stationary branes are domain walls and the time-dependent branes are FLRW-cosmologies, for which the fake supergravity and pseudo-supersymmetry formalism is by now well developed. However, the case of branes with co-dimension two is not included as these solutions depend on one *complex* coordinate rather than on one real coordinate.

An alternative, interesting, way to understand the existence of first-order equations for stationary and time-dependent brane solutions is given by the approach of mapping p-branes to (-1)-branes. The latter solutions are solely carried by the metric and scalar fields. It is then easy to observe that the scalar fields only depend on one coordinate and describe a geodesic motion on moduli space. In fact, for many cases this moduli space is a symmetric space, for which it is known that the geodesic equation of motion can easily be integrated to first-order equations (see for instance [36]). From this we expect that there exist BPS equations for all extremal and non-extremal black holes in theories which have a symmetric moduli space after reduction over one dimension [37].

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A. Construction of the effective action

As always one has to be careful when plugging an Ansatz into the action in order to obtain an effective action.

The action contains the following term

$$S_E = \int \mathrm{d}^4 x f(\psi_i)(F_{uz})^2 \,. \tag{A.1}$$

Here we denoted the other independent fields appearing in the action by ψ_i. For Einstein-Maxwell theory, we have $\{\psi_i\} = \{g_{\mu\nu}\}$ and $f(g_{\mu\nu}) = -\frac{1}{2}\sqrt{-g}g^{uu}g^{zz}$. We continue the discussion keeping ψ_i general. To calculate the contribution of this term to the field equations for the fields ψ_i, the vector field strengths have to be kept fixed when varying w.r.t. the other fields ψ_i. This gives

$$\frac{\delta S_V}{\delta \psi_i}\Big|_{F_{uz}} = \frac{\delta f}{\delta \psi_i} F_{uz} F_{uz} \tag{A.2}$$

What happens if we were to plug in the electrical Ansatz? The EOM are then solved by

$$F_{uz}(\psi_i) = f^{-1}(\psi_i) Q \,, \tag{A.3}$$

where Q is a constant (the electrical charge). Now the electric field strength becomes a function of ψ_i. This is in contrast to (A.1), where it had to be considered as an independent field in the action when calculating the EOM. Due to this fact, an effective action cannot be obtained by simply plugging the Ansatz into S_V. We have to flip the sign of S_V too

$$S_E^{\text{eff}} = -S_E(F_{uz}(\psi_i)) = \int \mathrm{d}^4 x(-f^{-1}Q^2) \,. \tag{A.4}$$

The reason is that now we keep Q, rather than the field strength F_{uz}, fixed while varying w.r.t. ψ_i. In particular, we see that we gain the correct contribution (A.2) to the EOM for ψ_i by varying the effective action

$$\frac{\delta S_E^{\text{eff}}}{\delta \psi_i}\Big|_{Q} = +f^{-2}\frac{\delta f}{\delta \psi_i} Q^2 = \frac{\delta S_E}{\delta \psi_i}\Big|_{F_{uz}} \,. \tag{A.5}$$

For a magnetic Ansatz we have

$$F_{z_1 z_2} = P \epsilon_{z_1 z_2} \,, \tag{A.6}$$

where we chose coordinates z_i on the slice Σ_k. The contribution to the action for a magnetic configuration is

$$S_M = \int \mathrm{d}^4 x g(\psi_i)(F_{z_1 z_2})^2 \,. \tag{A.7}$$

Plugging in this ansatz (A.6) will not change the EOM for the fields ψ_i, because now the field strength does not depend on the ψ_i.

References

[1] K.S. Stelle, *BPS branes in supergravity*, `hep-th/9803116`.

[2] M. Gutperle and A. Strominger, *Spacelike branes*, *JHEP* **04** (2002) 018 [`hep-th/0202210`].

[3] E.B. Bogomolny, *Stability of classical solutions*, *Sov. J. Nucl. Phys.* **24** (1976) 449 [*Yad. Fiz.* **24** (1976) 861].

[4] M.K. Prasad and C.M. Sommerfield, *An exact classical solution for the 't Hooft monopole and the Julia-Zee dyon*, *Phys. Rev. Lett.* **35** (1975) 760.

[5] E. Witten and D.I. Olive, *Supersymmetry algebras that include topological charges*, *Phys. Lett.* **B 78** (1978) 97.

[6] A. Ceresole and G. Dall'Agata, *Flow equations for non-BPS extremal black holes*, *JHEP* **03** (2007) 110 [`hep-th/0702088`].

[7] L. Andrianopoli, R. D'Auria, E. Orazi and M. Trigiante, *First order description of black holes in moduli space*, *JHEP* **11** (2007) 032 [`arXiv:0706.0712`].

[8] G. Lopes Cardoso, A. Ceresole, G. Dall'Agata, J.M. Oberreuter and J. Perz, *First-order flow equations for extremal black holes in very special geometry*, *JHEP* **10** (2007) 063 [`arXiv:0706.3373`].

[9] C.M. Miller, K. Schalm and E.J. Weinberg, *Nonextremal black holes are BPS*, *Phys. Rev.* **D 76** (2007) 044001 [`hep-th/0612308`].

[10] D.Z. Freedman, C. Núñez, M. Schnabl and K. Skenderis, *Fake supergravity and domain wall stability*, *Phys. Rev.* **D 69** (2004) 104027 [`hep-th/0312055`].

[11] A. Celi, A. Ceresole, G. Dall'Agata, A. Van Proeyen and M. Zagermann, *On the fakeness of fake supergravity*, *Phys. Rev.* **D 71** (2005) 045009 [`hep-th/0410126`].

[12] I. Papadimitriou, *Non-supersymmetric membrane flows from fake supergravity and multi-trace deformations*, *JHEP* **02** (2007) 008 [`hep-th/0606038`].

[13] K. Skenderis and P.K. Townsend, *Gravitational stability and renormalization-group flow*, *Phys. Lett.* **B 468** (1999) 46 [`hep-th/9909070`].

[14] M. Cvetič and H.H. Soleng, *Naked singularities in dilatonic domain wall space times*, *Phys. Rev.* **D 51** (1995) 5768 [`hep-th/9411170`].

[15] K. Skenderis and P.K. Townsend, *Pseudo-supersymmetry and the domain-wall /cosmology correspondence*, *J. Phys.* **A 40** (2007) 6733 [`hep-th/0610253`].

[16] K. Skenderis and P.K. Townsend, *Hidden supersymmetry of domain walls and cosmologies*, *Phys. Rev. Lett.* **96** (2006) 191301 [`hep-th/0602260`].

[17] D.S. Salopek and J.R. Bond, *Nonlinear evolution of long wavelength metric fluctuations in inflationary models*, *Phys. Rev.* **D 42** (1990) 3936.

[18] D. Bazeia, C.B. Gomes, L. Losano and R. Menezes, *First-order formalism and dark energy*, *Phys. Lett.* **B 633** (2006) 415 [`astro-ph/0512197`].

[19] W. Chemissany, A. Ploegh and T. Van Riet, *A note on scaling cosmologies, geodesic motion and pseudo-SUSY*, *Class. and Quant. Grav.* **24** (2007) 4679 [`arXiv:0704.1653`].

[20] K. Skenderis and P.K. Townsend, *Hamilton-Jacobi for domain walls and cosmologies*, *Phys. Rev.* **D 74** (2006) 125008 [`hep-th/0609056`].

JHEP04(2008)007

[21] P.K. Townsend, *Hamilton-Jacobi mechanics from pseudo-supersymmetry*, *Class. and Quant. Grav.* **25** (2008) 045017 [arXiv:0710.5178].

[22] P.K. Townsend, *From wave geometry to fake supergravity*, arXiv:0710.5709.

[23] B. de Wit and A. Zwartkruis, $\mathrm{SU}(2,2/1,1)$ *supergravity and* $N=2$ *supersymmetry with arbitrary cosmological constant*, *Class. and Quant. Grav.* **4** (1987) L59.

[24] K. Behrndt and M. Cvetič, *Time-dependent backgrounds from supergravity with gauged non-compact R-symmetry*, *Class. and Quant. Grav.* **20** (2003) 4177 [hep-th/0303266].

[25] E.A. Bergshoeff, J. Hartong, A. Ploegh, J. Rosseel and D. Van den Bleeken, *Pseudo-supersymmetry and a tale of alternate realities*, *JHEP* **07** (2007) 067 [arXiv:0704.3559].

[26] K. Skenderis, P.K. Townsend and A. Van Proeyen, *Domain-wall/Cosmology correspondence in AdS/dS supergravity*, *JHEP* **08** (2007) 036 [arXiv:0704.3918].

[27] S. Vaula, *Domain Wall/Cosmology correspondence in* $(AdS/dS)_6 \times S^4$ *geometries*, *Phys. Lett.* **B 653** (2007) 95 [arXiv:0706.1361].

[28] A. Van Proeyen and B. Vercnocke, *Effective action for the field equations of charged black holes*, *Class. and Quant. Grav.* **25** (2008) 035010 [arXiv:0708.2829].

[29] G.W. Gibbons, *Antigravitating black hole solitons with scalar hair in* $N=4$ *supergravity*, *Nucl. Phys.* **B 207** (1982) 337.

[30] G.W. Gibbons, M.B. Green and M.J. Perry, *Instantons and seven-branes in type IIB superstring theory*, *Phys. Lett.* **B 370** (1996) 37 [hep-th/9511080].

[31] E. Bergshoeff, A. Collinucci, U. Gran, D. Roest and S. Vandoren, *Non-extremal D-instantons*, *JHEP* **10** (2004) 031 [hep-th/0406038].

[32] M. Gutperle and W. Sabra, *Instantons and wormholes in Minkowski and (A)dS spaces*, *Nucl. Phys.* **B 647** (2002) 344 [hep-th/0206153].

[33] M. Kruczenski, R.C. Myers and A.W. Peet, *Supergravity S-branes*, *JHEP* **05** (2002) 039 [hep-th/0204144].

[34] H. Lü, S. Mukherji and C.N. Pope, *From p-branes to cosmology*, *Int. J. Mod. Phys.* **A 14** (1999) 4121 [hep-th/9612224].

[35] S. Ferrara, G.W. Gibbons and R. Kallosh, *Black holes and critical points in moduli space*, *Nucl. Phys.* **B 500** (1997) 75 [hep-th/9702103].

[36] D. Gaiotto, W.W. Li and M. Padi, *Non-supersymmetric attractor flow in symmetric spaces*, *JHEP* **12** (2007) 093 [arXiv:0710.1638].

[37] B. Janssen, P. Smyth, T. Van Riet and B. Vercnocke, work in progress.

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